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Trigonometric functions

All the formula for real trigonometric functions are valid for complex trigonometric function.

eg. $\cos z = \frac{e^{it} + e^{-it}}{2}$

$\sin z = \frac{e^{it} - e^{-it}}{2i}$

$\sin^2 z + \cos^2 z = 1$ $\sin(z_1 + z_2) = \sin z_1 \cos z_2 + \cos z_1 \sin z_2$ and so on

Hyperbolic function - Hyperbolic cosine of z is written as $\cosh z$ and is defined as $\frac{e^z + e^{-z}}{2}$

i.e. $\cosh z = \frac{e^z + e^{-z}}{2}$

Similarly $\sinh z = \frac{e^z - e^{-z}}{2}$

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Properties of Hyperbolic functions

(a) $\sinh z$ and $\cosh z$ are periodic functions with imaginary period $2\pi i$

(b) $\cosh z$ is an even function while $\sinh z$ is an odd function.

c) $\sinh 0 = 0 \quad \operatorname{cosech} 0 = 1 \quad \tanh 0 = 0$

d) $\sin(iz) = i\sinh z$,
$\cos(iz) = \cosh z$
$\tan(iz) = i\tanh z$
Also $\sinh(iz) = i\sin z$, $\cosh(iz) = \cos z$
$\tanh(iz) = i\tan z$.

Important Results on Hyperbolic functions

(a) $\cosh^2 z - \sinh^2 z = 1$
$\operatorname{sech}^2 z + \tanh^2 z = 1$
$\coth^2 z - \operatorname{cosech}^2 z = 1$